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HEAT TRANSFER THROUGH SOLID DISPERSE SYSTEMS

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Heat transfer through different model solid disperse systems is examined. Functional relations permitting calculation of the effective heat conductivity of the system are established.

Let a disperse system be represented by a mechanical mixture of two components. In particular, one of the components may be a gas; then the disperse system will be a porous solid. Let us examine two models of a disperse system, which we will call the model with closed pores and the model with interconnecting pores.* In the first model the basic material (component 1) is interspersed with particles of a foreign material (component 2), which do not interconnect. In the second model the particles of foreign material are interconnecting. Below we examine models of disperse systems with long-range order, i.e., systems in which the inclusions are orderly distributed and their dimensions are approximately the same. Also let the inclusions be either solid or gaseous, i.e., moist materials are not considered.

It will be shown below that the effective heat conductivity of such a system depends on the heat conductivity of component 1, the heat conductivity of component 2, and on the volume concentration n of the inclusions ($n = V_2/V$).

If component 2 is a gaseous inclusion, then the volume concentration coincides with the porosity p of the system, i.e.,

$$n = p = V_2/V = 1 - \gamma'/\gamma. \tag{1}$$

The effective heat conductivity of a solid disperse system is conveniently found in the form

$$\lambda/\lambda_1 = f(n, \ \lambda_2/\lambda_1), \tag{2}$$

in which λ_1 characterizes heat transfer through the solid skeleton allowing for both conductive λ_{1c} and radiant λ_{1r} transfer components, and λ_2 characterizes heat transfer through component 2. If component 2 consists of gaseous inclusions, then the coefficient λ_2 takes into account both the molecular λ_{2m} and the radiant λ_{2r} components.

The dependence of λ_1 and λ_2 on their characteristic parameters must be established separately in the form

$$\lambda_1 = f_1(\lambda_{1c}, \ \lambda_{1r}), \quad \lambda_2 = f_2(\lambda_{2c}, \ \lambda_{2r}) \quad \text{or} \quad \lambda_2 = f_3(\lambda_{2m}, \ \lambda_{2r}). \tag{3}$$

Equations (2) and (3) quantitatively describe the effective heat conductivity of the system.

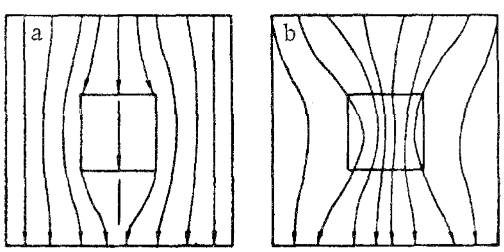


Fig. 1. Lines of heat flow through unit cell: a) $\nu < 1$; b) $\nu > 1$.

In order to develop the form of functional relation (2), it is necessary to formulate more precisely the geometrical features of the model disperse system.

We will examine first the model with closed pores. It is possible to propose a number of variants of the mutual spacing and form of the inclusions in a system with long-range order, but we will examine the simplest: the centers of the cubic inclusions are distributed along straight lines. The functional relation (2) for such a model was established in [1–4]. Since analysis of these papers showed that the most correct derivation of the formula is that given by V. I. Odelevskii, we will examine his basic ideas concerning the choice of relation (2).

In a system with long-range order, it is possible to distinguish the smallest volume (unit cell), whose effective heat conductivity coincides with the effective heat conductivity of the disperse system. The unit cell is divided into characteristic parts, whose heat resistance is easy to calculate; further, the total resistance of the cell is calculated by the method of electrothermal analogy. Different ways of dividing the unit cell into its characteristic parts are possible. Different divisions lead to different formulas, which give different numerical values of the effective heat conductivity. Odelevskii proposed the following form for relation (2):

$$\frac{\lambda}{\lambda_1} = 1 - n\left(\frac{1}{1-\nu} - \frac{1-n}{3}\right), \quad \nu = \frac{\lambda_2}{\lambda_1}, \tag{4}$$

*For brevity, component 2 is called a pore; in particular, if component 2 is a gas, then the concept "pore" conforms with the usual definition.

which leads to intermediate numerical results for λ. This is attained by a particular choice of the characteristic parts of the unit cell.

In establishing relation (4), Odelevskii implicitly assumed that the heat flow lines in the unit cell were not distorted. Actually, when $\lambda_1 \neq \lambda_2$ the heat flow lines are distorted and take the form shown in Fig. 1. In calculating the effective heat conductivity of the system this can lead to errors, the magnitude of which must be estimated. With this aim we carried out a special investigation on an electric analog computer. The investigation was carried out at five values of n = 0, 0.216, 0.512, 0.729, and 1, and four different values of ν = 0, 0.3, 0.5, and 1.

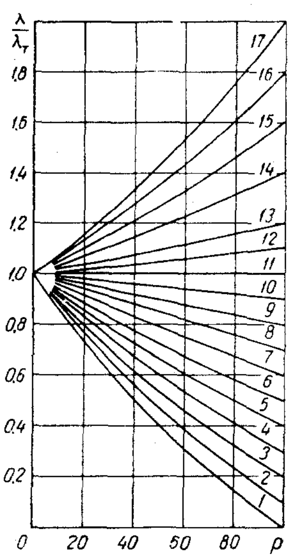
In the table the results obtained using the electric analog computer are compared with the results calculated from (4).

Values of Effective Heat Conductivity

n	0.216			0.512			0.729		
ν	0	0.3	0.5	0	0.3	0.5	0	0.3	0.5
λ	0.696	0.815	0.874	—	0.576	0.700	0.197	0.455	0.625
λ_1	0.707	0.816	0.876	0.388	0.596	0.722	0.200	0.457	0.619

For n = 0 and 1 and ν = 1 the values of the effective heat conductivity obtained from (4) are $\lambda = \lambda_1$, which is obvious. Comparison of the results obtained shows that they differ by no more than 4%, which lies in the limits of instrumental error of the analog computer.

It is interesting to examine the form of (2) for models with inclusions of different shapes. For spherical inclusions relation (2) was established in studies of the effective dielectric constant or electrical conductivity of two-component systems [6–8]. The results calculated from the formulas given in these papers and from (4) virtually coincide.



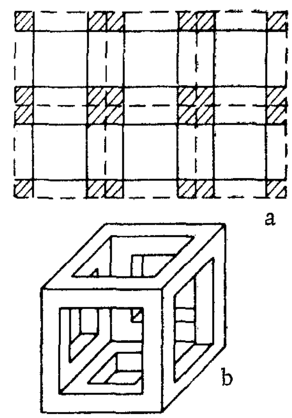


Fig. 3. Model system with interconnecting pores: a) diagram of model; b) unit cell.

Fig. 2. Dependence of the effective heat conductivity of a system with closed pores on the concentration and heat conductivity of the skeleton λ_1 and inclusions λ_2 according to Eq. (4): 1) ν = 0; 2) 0.1; 3) 0.2; 4) 0.3; 5) 0.4; 6) 0.5; 7) 0.6; 8) 0.7; 9) 0.8; 10) 0.9; 11) 1.0; 12) 1.1; 13) 1.2; 14) 1.4; 15) 1.6; 16) 1.8; 17) 2.0.

The results of the investigation indicate that cubic and spherical inclusions give practically the same values for the effective heat conductivity of the system. Further, distortion of the flow lines in the unit cell has little influence on the effective value of the heat conductivity. Finally, of the numerous formulas for the heat conductivity of a system with closed pores, the best is that proposed by Olevskii. In Fig. 2 Olevskii's relation (4) is given in graphical form.

The models examined above relate to systems with closed cubic or spherical pores. We failed to discover in the literature an expression for the heat conductivity of systems with interconnecting pores. A model of such a system is represented in Fig. 3a, while Fig. 3b shows the unit cell. We denote by l the basic dimension of a pore (second component), and by L the outside dimension of the unit cell, $h = 2\Delta$ is the thickness and width of a rib of the skeleton,

V and V_1 are the total volume and the volume of the skeleton in the unit cell, and V_2 is the volume of the second component.

We will establish a relation between the volume concentration n and the ratio h/l. Since the unit cell is symmetrical, it is sufficient to examine a quarter of it, the volumes of which may be denoted by V', V_1', and V_2'. It is obvious that

$$V' = \frac{1}{4}L^3, \quad V_1' = \Delta^2(3L - 4\Delta).$$

Then

$$\frac{V_1'}{V'} = \frac{V_1}{V} 4\left(\frac{\Delta}{L}\right)^2\left(3 - 4\frac{\Delta}{L}\right), \quad (5)$$

$$\frac{\Delta}{L} = \frac{h}{2(l+h)} = \frac{\Delta/l}{1+2\Delta/l}.$$

We set $\Delta/L = x$; then (5), with account for (1), can be written

$$4x^3 - 3x^2 + k = 0, \quad k = (1-n)/4. \quad (6)$$

Solving (6) for x in the usual way and noting that

$$h/l = x/(0.5 - x), \quad (7)$$

we obtain the relation $n = n(n/l)$ (for n = 0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, and 1.0; h/l = ∞, 4.21, 2.47, 1.75, 1.29, 0.846, 0.700, 0.567, 0.396, 0.244, and 0).

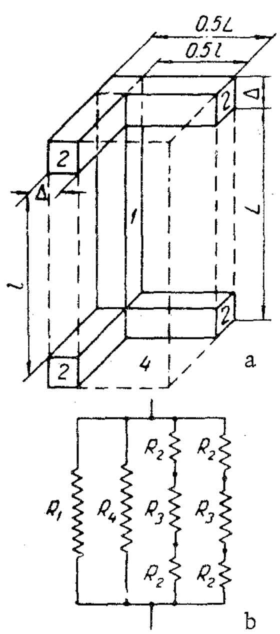


Fig. 4

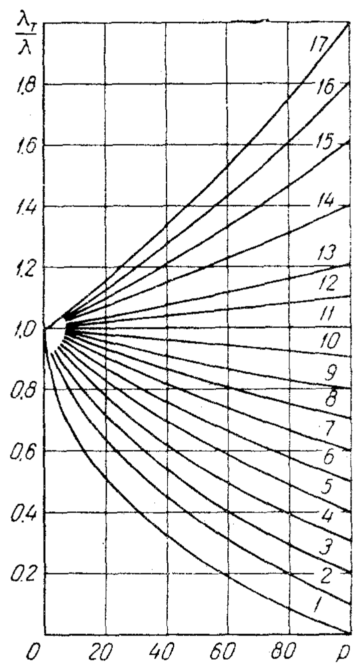


Fig. 5

Fig. 4. A quarter of the unit cell (a) and heat resistance connection diagram (b).

Fig. 5. Relation between the effective heat conductivity of a system with interconnecting pores and the concentration and heat conductivity of the skeleton λ_1 and the inclusions λ_2 according to (11). 1–17) see Fig. 2.

We will now calculate the heat resistance of the separate elements of the cell, assuming that the heat flow lines are parallel to each other and parallel to the vertical generators of the cell. We divide one quarter of the unit cell into four parts (Fig. 4a) and calculate their heat resistance:

$$R_1 = \frac{L}{\lambda_1\Delta^2}, \quad R_2 = \frac{2}{\lambda_1 l}, \quad R_3 = \frac{2}{\lambda_2\Delta}, \quad R_4 = \frac{4L}{\lambda_2 l^2}. \quad (8)$$

The effective heat resistance of the quarter cell

$$R_{eff} = 4/\lambda L. \tag{9}$$

The heat resistance connection diagram is given in Fig. 4b, from which it follows that

$$\sigma_{eff} = \sigma_1 + \sigma_2 + 2\sigma', \quad R' = 2R_2 + R_3, \quad \sigma_i = 1/R_i. \tag{10}$$

From (8)–(10) we obtain

$$\frac{\lambda}{\lambda_1} = \left(\frac{h}{L}\right)^2 + \nu\left(1 - \frac{h}{L}\right)^2 + 2\nu\frac{h}{L}\left(1 - \frac{h}{L}\right)\left[1 - \frac{h}{L}(1 - \nu)\right]^{-1}, \tag{11}$$

$$L = l + h, \quad \frac{h}{L} = \frac{h/l}{1 + h/l}, \quad \nu = \frac{\lambda_2}{\lambda_1}. \tag{12}$$

Equation (11) gives the structure of the functional relation (2) for disperse systems with interconnecting pores. Assigning n, we find from the data presented above the ratio h/l, from (12) we calculate h/L and, knowing ν, from (11) we determine the effective heat conductivity of the system.

Thus, for model disperse systems with closed pores the functional relation (2) is represented analytically by (4) and graphically by Fig. 2; for models with interconnecting pores this relation is given by (11) and by Fig. 5.

Let us now find the structure of (3) for porous systems (component 2 a gas). Let δ be the pore size, and S its corss-sectional area; then the effective σ_{2eff} conductivity of the pore is equal to the sum of the molecular σ_{2m} and radiant σ_{2r} conductivities, and since

$$\sigma_{2i} = \lambda_{2i} S/\delta \quad (i = \text{eff, m, r}),$$
$$\lambda_2 = \lambda_{2m} + \lambda_{2r}. \tag{13}$$

The molecular heat conductivity can be calculated from the equation [9]

$$\lambda_{2m} = \frac{\lambda_0}{1 + B/H\delta}, \quad B = 760\frac{4k}{k+1}(\mathrm{Pr})^{-1}\frac{2-a}{a}\Lambda. \tag{14}$$

The radiant heat conductivity in the pore can be calculated from the equation [5, 6]

$$\lambda_{2r} = 2\varepsilon^2 C T^3 \delta. \tag{15}$$

If the material of the skeleton is opaque to radiation, the heat conductivity λ_1 is completely determined by conductive transfer, i.e., $\lambda_1 = \lambda_{1c}$. For a transparent skeleton it remains necessary to find the structure of (3).

We note that the derived functional relations (2), (4), and (11) can be adapted to solid disperse phases with inclusions consisting of a mechanical mixture of many materials. Let, for example, an inclusion consist of two different components. First it is necessary to consider the inclusion as one material and to use (4) or (11) to determine the effective heat conductivity of the system; subsequently, however, the same relations are applied only to the inclusions and the structure of the heat conductivity of the two-component inclusions is found, and so on.

Notation

λ—effective heat conductivity; λ_1 and λ_2—heat conductivity of components 1 and 2; λ_0—heat conductivity of gas; V_2—volume occupied by component 2; V—total volume of system; γ'—volume weight of solid; γ—specific weight of component; k—ratio of specific heats of gas at constant pressure and volume; a—accommodation coefficient; Λ—mean free path of molecule under normal conditions; H—gas pressure; ε—emissivity of pore surface; $C = 5.67 \times 10^{-8}$ W/m·deg—Stefan-Boltzmann constant; T—mean absolute temperature of material.

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